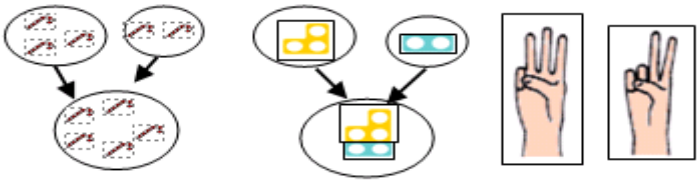
Livingstone Primary School
Calculations Progression 2014

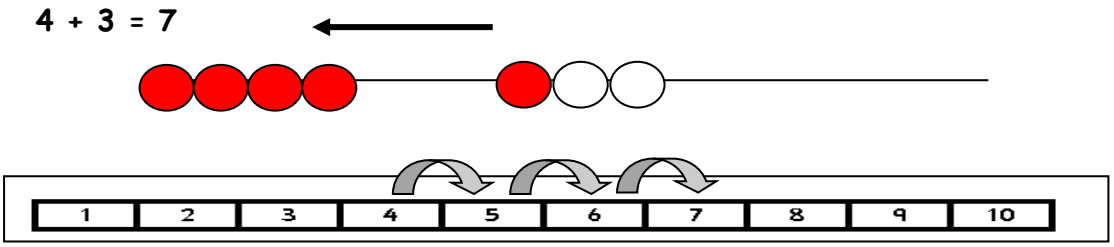
ADDITION
Y1

Expectation: Read, write and interpret statements involving +,-,= signs.
Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations and missing number problems such as 7=[] – 9
Represent and use number bonds and related subtraction facts within 20
Add and subtract one-digit and two-digit numbers to 20, including 0

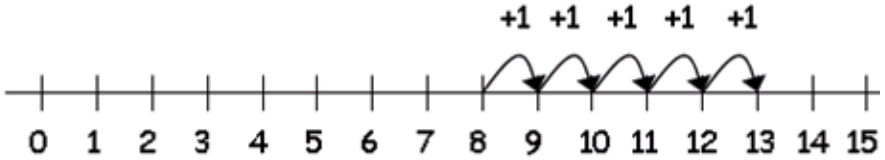
- Use developing mathematical ideas and methods to solve practical problems involving **counting** and comparing in a real or role play context using Models and Images to support learning.



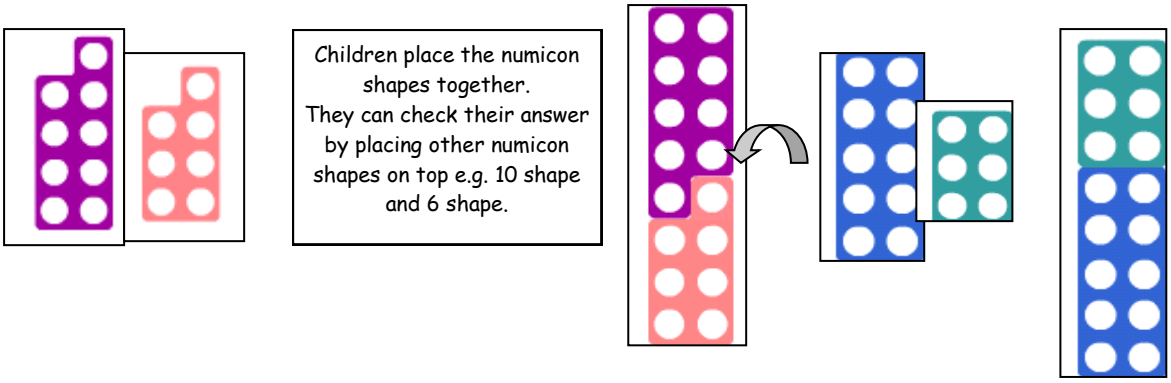
- Children develop ways of recording calculations using numicon, pictures, fingers, number tracks and bead strings etc. **3 + 2 = 5**
- Say how many there are when two groups of objects are combined to make a total; extend to three groups using imagery of combining sets to develop understanding.
- Begin to use vocab. involved in addition (more, and, add, make, sum, total, altogether, score… one more, two more…, how many more to make….? How many more is… than…?). Use related vocab. and symbols to describe and record addition number sentences.
- Can answer questions such as:
  What is this number? Which is more: 4 or 7? What number: comes after 10? is one more than 5, 12?
- Begin to relate addition to counting on. Encourage children to count on from the larger number.
- Recognise that addition can be done in any order to do mental calculations more efficiently. Put the larger number first and count on in ones (stepping), including beyond 10, using the bead string or labelled number line.



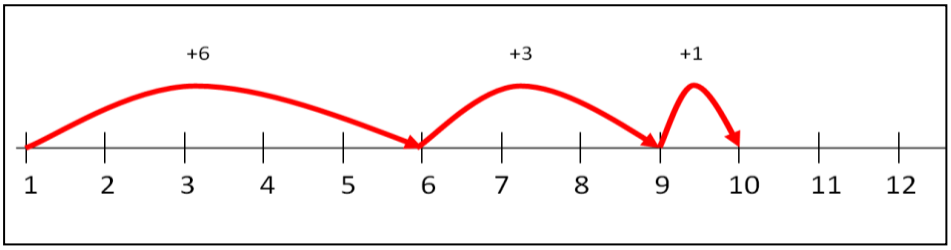
- Children should then begin to use a numbered line to support their own calculations to count in ones.



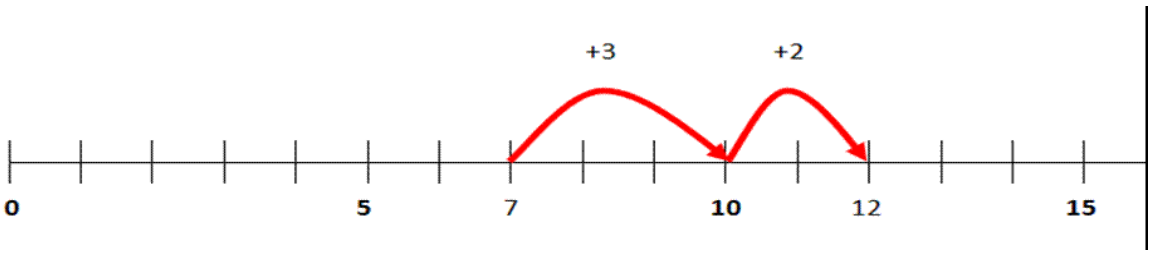
- Begin to interpret situations as addition calculations and explain reasoning, for example, can answer questions such as: *Lisa has 9 pens and Tim has 7 pens. How many pens do they have altogether?*



- Understand that more than two numbers can be added together, e.g. explore three jumps to 10 (or any other suitable number). 6 + 3 + 1 = 10



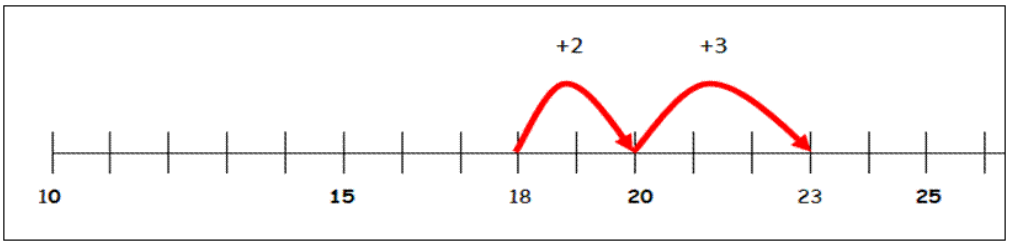
- When children are reliably jumping begin to remove some of the support provided by the fully marked and labelled number line. Add a pair of numbers mentally by:
jumping and **bridging** through 10



Y2

Expectation: Solve problems with addition & subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; applying their increasing knowledge of mental and written methods.
Recall and use addition and subtraction facts to 20 fluently, derive and use related facts up to 100.
Use number facts to solve problems
Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; adding three one-digit numbers.
Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.

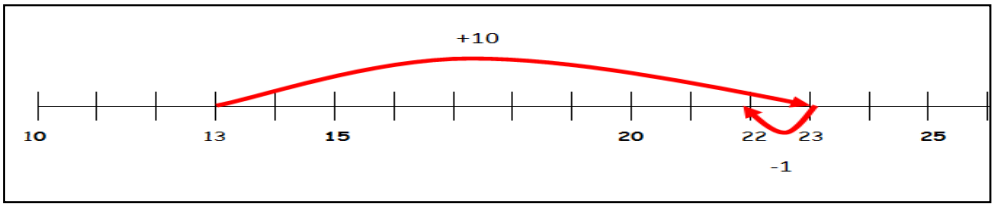
- When children are reliably jumping begin to remove some of the support provided by the fully marked and labelled number line. Add a pair of numbers mentally by:
Jumping and **bridging** through 20.



Rounding and adjusting: Add a multiple of 10 and adjust by 1

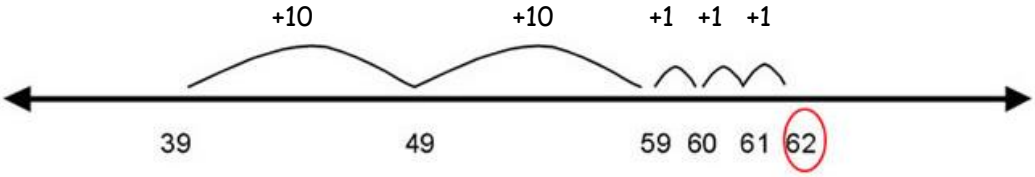
13 + 9

13 +10 - 1

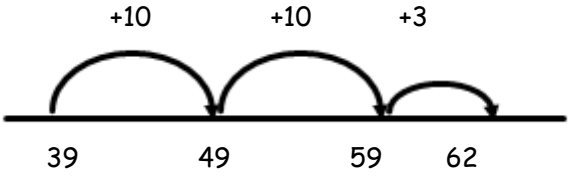


- Children start to use an empty number line partition the 'tens' first as the most significant digit on an empty number line. Numicon number rods can be used to support this initially and give a visual image of what is occurring. Initially count on in 10s and then 1s.

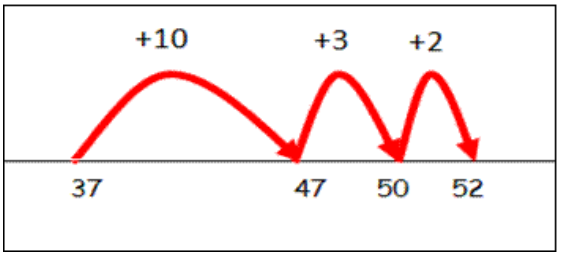
39 + 23 = 62



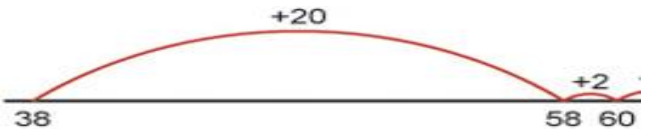
Then becoming more efficient by adding the ones in one jump.



37 + 15 = 52 Bridging through ten can help children become more efficient.



Followed by adding the tens in one jump and the units in one jump. eg. 38 + 22= 60



Children are continuing to add using their mental methods asking themselves the question:
Which mental method should I use? They will look at the numbers in the calculation in order to make a decision about whether to use:

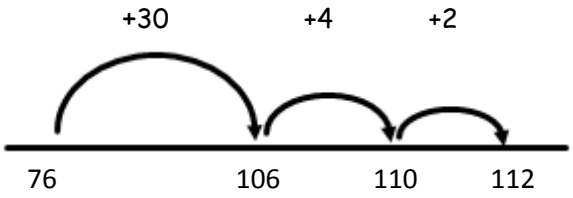
Partitioning Reordering Using near doubles

Compensation (rounding and adjusting)

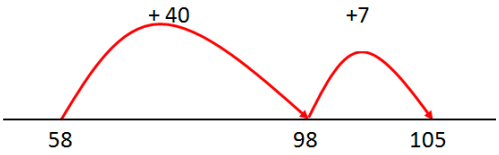
It is important that children learn number facts concurrently with written methods as they progress through the school. Children will struggle to access this method without a secure understanding of the number facts.

As the numbers continue to increase, children use their understanding of place value and number facts to use a number line flexibly.

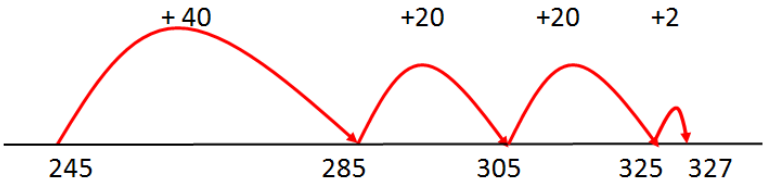
Eg. 76+36



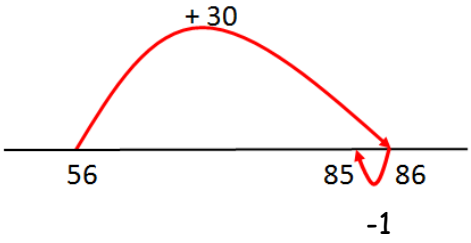
Eg. 47 + 58 = 105



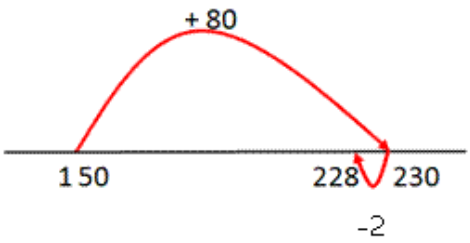
Eg. 245 + 82 = 327



Eg. 56 + 29 = 85



Eg. 149 + 79 = 228



Y3

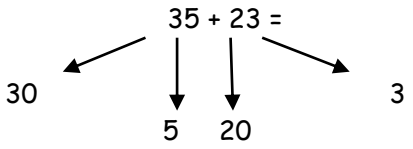
Expectation: Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Add units, tens or hundreds to 3-digit numbers mentally.

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

Estimate the answer to a calculation and use inverse operations to check answers.

- Children's mental methods of calculation must be practised and secured alongside their learning and development towards a compact written method.
- For children to be able to use written methods for addition they need to be able to:
    - Recall addition pairs to 9+9 and complements to 10.
    - Add mentally a series of one digit numbers
    - Add multiples of 10 or of 100 using related addition facts and their knowledge of place value.
    - Partition two digit and three digit numbers into multiples of 100, 10 and 1 in different ways.
- As a brief introduction to columnar addition, children need to explore adding through partitioning so that they understand what is happening in columnar addition.



30 + 20 = 50
5 + 3 = 8

50 + 8 = 58

Informal Expanded Method

This leads children to the more compact written method developing an understanding of its structure and efficiency. Numbers are still added up mentally.

Two and three digit addition:

This method should only be introduced when adding a 3 digit to a 2 digit number. Adding two 2 digit numbers should be added mentally.

	1	6	8	
+		5	7	
		1	5	(8 + 7)
	1	1	0	(60 + 50)
	1	0	0	
	2	2	5	

Place Value counters and boards MUST be used whilst recording in order to develop understanding and efficiency. This MUST follow a pattern of *Action-Record, Action-Record*

```
  1 6 8
+   5 7
  1 5 ( 8 + 7 )
1 1 0 ( 60 + 50 )
  1 0 0
-------
  2 2 5
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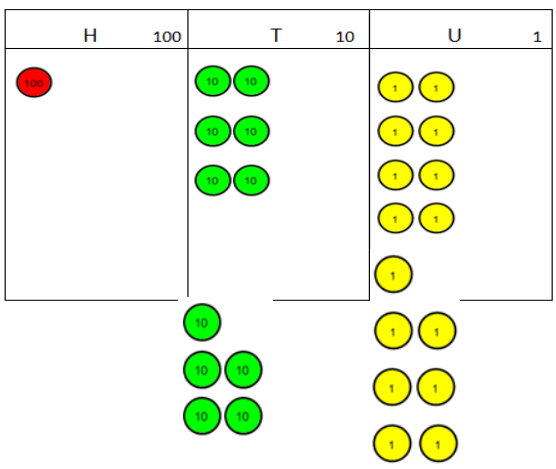


The agreed vocabulary to be used across the MCSP partnership is to '**change**' ten 1s for a 10

The amount of time that should be spent teaching the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Compact Method

All of the previous strategies lead up to the column method as one reliable method the children can apply when mental methods are not appropriate. The method doesn't change but the recording is reduced.
Children may become more efficient by first using place value counters and grids.

Column headings should be included to remind children of the value of a digit during the introduction to this method. Children should learn how to use column addition without 'crossing 10' to begin with.

E.g.

	H	T	U
	1	2	3
+		5	4
	1	7	7

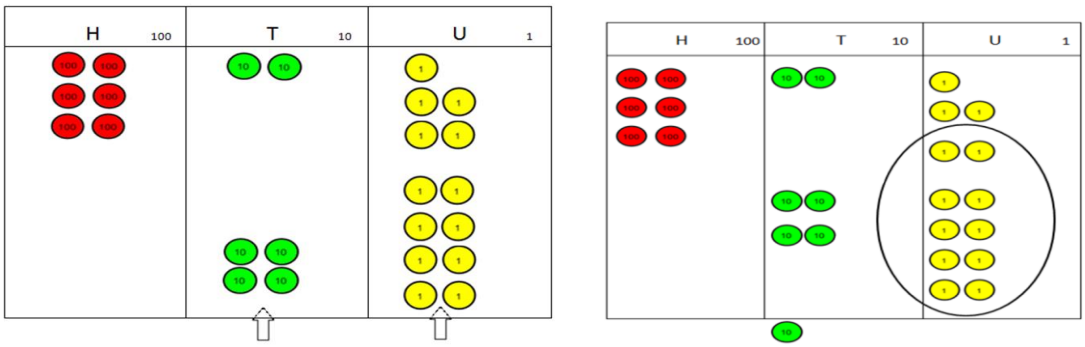
Children should always be taught to begin with the smallest value (Eg. ones)

Children should extend this method to crossing 10 and use the carrying method. In this method recording is reduced further. Carried digits would be recorded below the line. The words **change** ten 1s for a ten or ten 10s for a hundred and NOT carry the one. This is the agreed terminology across the MCSP cluster.

This method should still be started along with the place value grids and counters.

E.g.

	H	T	U
	6	2	5
+		4	8
	6	7	3
		1	



In Y3 children should be using the compact method to add up to 2 three digit numbers.

Y4

Expectation: Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.
Estimate and use inverse operations to check answers to a calculation.

- Children's mental methods of calculation must be practised and secured alongside their learning and development towards a compact written method.

Compact method

In Y4 children should be using the compact method to add four digit numbers and a range of decimal numbers.

	T	U	.	t	h
	3	5	.	5	7
+	2	4	.	9	8
	6	0	.	5	5
	1	1		1	

Y5

Expectation: Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
Add and subtract numbers mentally with increasingly large numbers.

- Children's mental methods of calculation must be practised and secured alongside their learning and development towards a compact written method.

Compact Method

The compact method can also be used to add two or more decimal fractions with at least 4 digits and 2 or 3 decimal places for money/measures (Year 5 objective) E.g. time to thousandths of a second.

For example, Mo Farrah ran the first lap of his 10Km race in 47.142 seconds and the second in 49.295 seconds. How long did it take him to run the first two laps?

```
  47.142
  49.295
 ---------
+ 96.437 seconds
 ---------
  1   1
```

Children should know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts. E.g. 43.2m + 2900cm.

Y6

Expectations: Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Perform mental calculations, including with mixed operations and large numbers

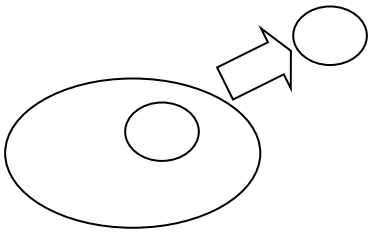
Once learned, the column method is quick, efficient and reliable. This method compliments the written method for subtraction, allows children to solve one and two step problems in a range of practical areas and is the agreed final stage for the MCSP collaboration, ensuring continuity as children progress into secondary education.

SUBTRACTION

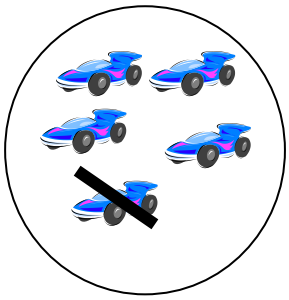
Y1

Expectation: Read, write and interpret statements involving +,-,= signs.
Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations and missing number problems such as 7=[] – 9
Represent and use number bonds and related subtraction facts within 20
Add and subtract one-digit and two-digit numbers to 20, including 0

Children should first be encouraged to develop a mental picture of the number system to use for calculation. This may involve the use of pictures, visual resources and/or Numicon to develop ways of recording calculations. Children should be introduced to range of terms for 'subtract' when engaged in play and encouraged to use numbers to write emergent calculations. They need to have a visual image of the process of subtraction. They will use the Numicon covers to cover the appropriate section of a piece and see how much is left.



For example: Children working with a hoop and cars – I have five cars and one goes into the garage, how many are left? Children can take the appropriate number away – photographic recording would show this process as a visual way of recording. This would transfer to pictorial recording where a child would have a set of objects and would record by crossing out the appropriate number.



5 – 1 4
5 – 1 = 4

Solve problems in everyday life in the classroom, or in role play. Make decisions about what to do. Explain orally and where appropriate, can answer questions such as:

Which is less: 4 or 7? What number: comes before 10, is one less than 5, 12 etc.

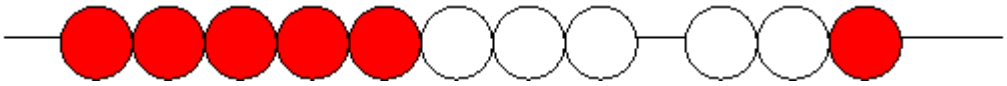
Use bead strings, number tracks and mathematical imagery and pattern within number e.g. numicon shapes to locate numbers quickly and is beginning to use vocab involved in subtraction such as : take away, leave, how many are left? How may have gone? One less, two less, how many less is …. than …. ? Difference between.

Say how many are left when some are taken away by counting how many are left or by counting back from the number. Find out how many have been removed or how many more will make a given number by counting up to the larger number.

Uses related vocab and symbols to describe and record subtraction number sentences.

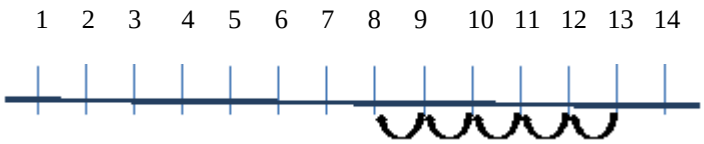
Subtracts one digit numbers from one and two digit numbers using the bead string or labelled number line by **stepping**.

11 - 3 = 8



Children begin to use labelled number lines as well as visual representations/resources, supported by teacher modelling of how to use the number line. EG. 5 – 3 = 2

13 – 5 = 8



The number line should also be used to show that 5 – 3 means the 'difference between 5 and 3' or the 'difference between 3 and 5' and how many jumps they are apart.

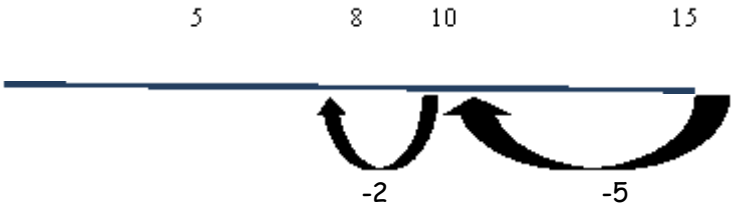
Children should then use numbered lines to support their own calculations; using a numbered line to count back in ones. EG. 12 – 3 = 9

Begin to interpret situations as subtraction calculations. For example can answer questions such as:
There are 18 bean bags. Jodie takes 5. How many bean bags are left?

Only when children are reliably jumping begin to remove some of the support provided by the fully marked and labelled number line.

Subtract a pair of numbers mentally by jumping and bridging through 10.

Bridge through a multiple of 10. 15—7 = 8



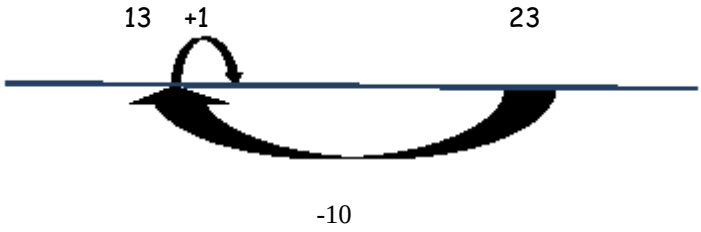
Y2

Expectation: Solve problems with addition & subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; applying their increasing knowledge of mental and written methods.
Recall and use addition and subtraction facts to 20 fluently, derive and use related facts up to 100.
Use number facts to solve problems
Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; adding three one-digit numbers.
Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.

When children are reliably jumping begin to remove some of the support provided by the fully marked and labelled number line. Numicon rods can be used to support this initially to create a visual image of the process.

Rounding and adjusting: Subtract a multiple of 10 and adjust by 1 (rounding and adjusting)

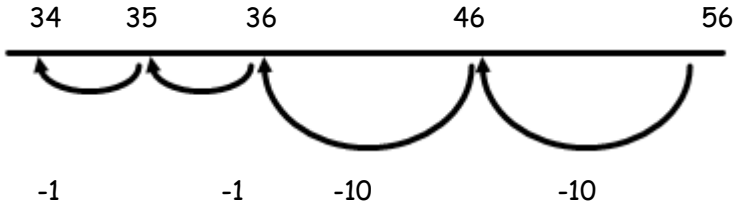
23—9 = 23—10 + 1 = 14



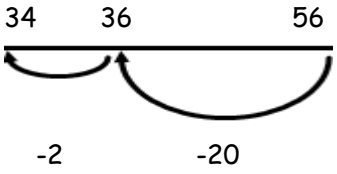
With developing recall of number facts, steps in mental subtraction can be recorded on an empty number line.

Children will then begin to use empty number lines to support calculations.

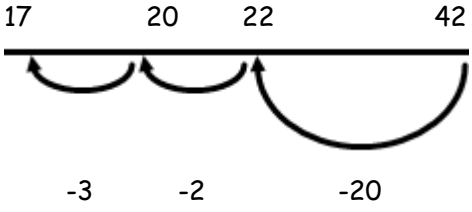
First counting back in tens and ones. EG. 56 – 22 = 34



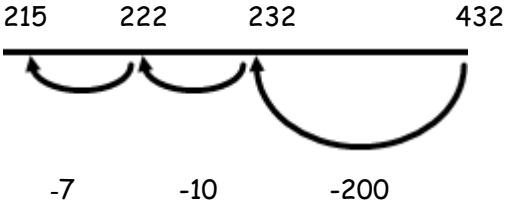
Then gradually becoming more efficient by subtracting the tens in one jump and the ones in another jump.



Bridging through 10 can help children to become more efficient.
42 – 25 = 17

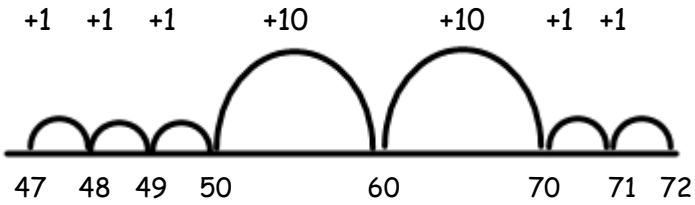


This can be extended to subtraction of 3 digit numbers.
432—217 = 215



If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on. Discuss with the children whether to subtract by counting on or taking away depending on which is the most efficient for the numbers in the calculation.

72-47 = 25



MENTAL CALCULATIONS (On-going)

Mental recall of number bonds: 10 — 6 = 4 20 — 17 = 3 17 — = 11

Find a small difference by counting up:

82—79 = 3

Counting on and back in repeated steps of 1, 10, 100, 1000:

86 - 52 = 34 (by counting back in tens and then in ones)
460—300 = 160 (by counting back in hundreds)

Rounding and adjusting, subtracting the nearest multiple of 10,100 and 1000 and adjust

24 - 19 = (24 - 20) + 1 = 5 458 - 71 = (458 - 70) - 1 = 387

Use the relationship between addition and subtraction: 36 + 19 = 55 19 + 36 = 55
55 -19 = 36 55 – 36 = 19

Y3

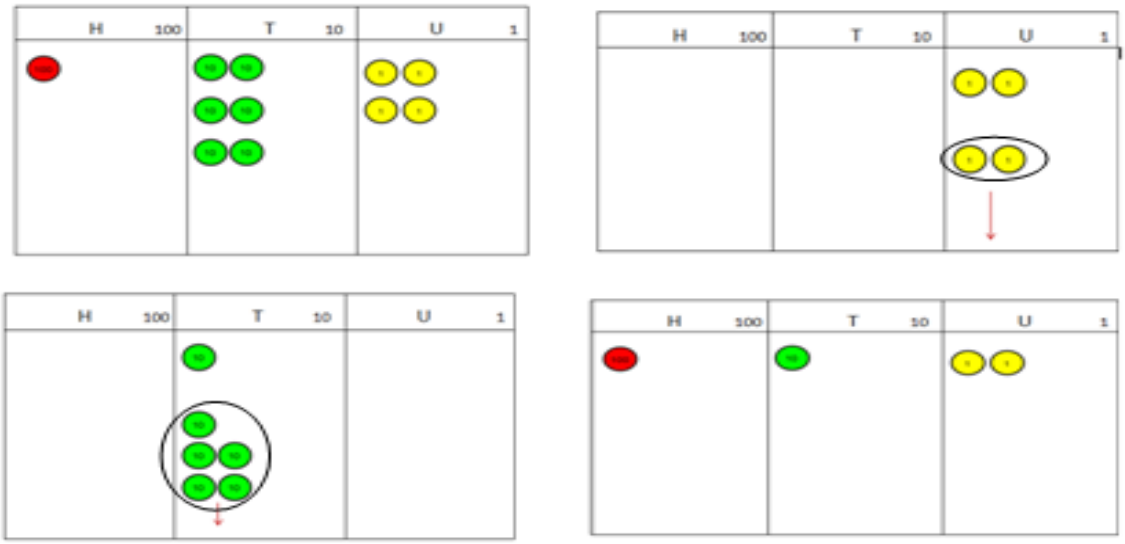
Expectation: Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction
Add units, tens or hundreds to 3-digit numbers mentally.
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.
Estimate the answer to a calculation and use inverse operations to check answers.

- Children's mental methods of calculation must be practised and secured alongside their learning and development towards a compact written method.
- For children to be able to use written methods for subtraction they need to be able to:
    - Recall all addition and subtraction facts to 20.
    - Subtract multiples of 10 or of 100 using related subtraction facts and their knowledge of place value.
    - Partition two digit and three digit numbers into multiples of 100, 10 and 1 in different ways.

Informal Expanded Method

This leads children to the more compact written method developing an understanding of its structure and efficiency. Two and three digit subtraction: Start with subtracting the ones, then the tens etc. This expanded method can be supported by the place value counters and grids to enable children to see what this method involves. When using this approach, children MUST use the *Action-Record.*

164
− 52
2
10
100
112



Only do the Informal Expanded Method with no "**change**" occurring. Any calculations involving "**changing**" a ten for 10 ones or a hundred for 10 tens will be left until the children start the compact method.

Compact Method

All of the previous strategies lead up to the column method as one reliable method the children can apply when mental methods are not appropriate. In the initial stages the place value board and counters still need to be used in order that the children establish an understanding of what is happening to the numbers. The method doesn't change but the recording is reduced.

Column headings should be included to remind children of the value of a digit during the introduction to this method. Children should learn how to use column subtraction without 'crossing 10' to begin with.

E.g. H T U
1 2 3
- 1 2
1 1 1

Children should always be taught to begin with the smallest value (EG ones)

Using similar methods, children will:

- Subtract numbers with different numbers of digits;
- Begin to subtract sums of money, with or without adjustment from pence to pounds;
- Know that decimal points should line up under each other, particularly when subtracting mixed amounts. (EG. £3.67 - 79p).

Children should extend this method to **"change 10"** and use the changing method. The term "changing" is the agreed terminology across the MCSP partnership. In this method recording is reduced further. Changed digits would be recorded above the digit. Initially it can help to cross out the original number and replace the whole number. The words change ten or hundred would be used and NOT change the one.

H6T U
1 ~~7~~ 16
- 4 7
1 2 9

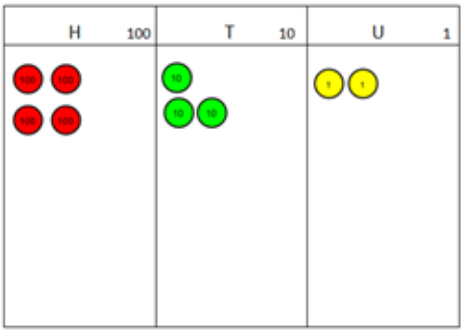
When initially introduced it may be helpful to cross out the 6 in the ones column and replace it with a 16.

This method can also be supported through the use of place value counters and grids. EG. 432-217=

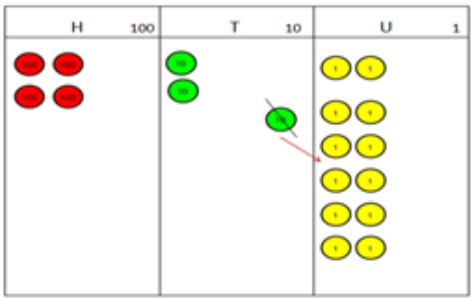
One adjustment, tens to units

432– 217=

432–217 = 215



2 subtract 7 is not possible so we have to exchange one ten for ten units:



		2	12
	4	~~3~~	~~2~~
-	2	1	7
	2	1	5

The Y3 expectation is that children are able to subtract 2 and 3 digit numbers using the compact method.

Y4

Expectation: Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.
Estimate and use inverse operations to check answers to a calculation.

- Children's mental methods of calculation must be practised and secured alongside their learning and development towards a compact written method.

Compact method

In Y4 children should be using the compact method to subtract four digit numbers and a range of decimal numbers. The recording is further reduced as the "changed" number is simply added to the original one.

```
  T U . t h
  3 4 5 . 15 7
- 2 4 . 9 7
  1 0 . 6 0
```

Y5

Expectation: Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
Add and subtract numbers mentally with increasingly large numbers.

- Children's mental methods of calculation must be practised and secured alongside their learning and development towards a compact written method.

Compact Method

The compact method can also be used to subtract two or more decimal fractions with at least 4 digits and 2 or 3 decimal places for money/measures (Year 5 objectives). For example, time to thousandths of a second.
For example:
Mo Farrah ran the first two laps of his 10Km race in 96.437seconds. The second lap took 49.295 seconds. How long did it take him to run the first lap?

```
  8 16  3 13
  9 6 4 3 7
-   4 9.2 9 5
  ---------
    4 7.1 4 2
  ---------
```

Children should know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts. E.g. 43.2m - 2900cm.

Y6

Expectations: Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Perform mental calculations, including with mixed operations and large numbers

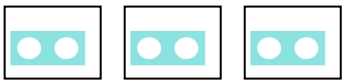
Once learned, the column method is quick, efficient and reliable. This method compliments the written method for addition, allows children to solve one and two step problems in a range of practical areas and is the agreed final stage for the MCSP collaboration, ensuring continuity as children progress into secondary education.

MULTIPLICATION

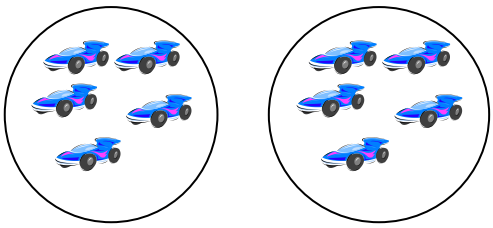
Y1

Expectations: Count in multiples of twos, fives and tens.
Solve one-step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children will first approach multiplication in play and problem solving. Children will experience equal groups of objects. They will work on practical problem solving activities involving equal sets or groups. This may involve the use of pictures, Numicon, visual resources and/or other resources to develop ways of recording calculations.



Children should be introduced to range of terms for 'multiply' (such as 'lots of' and 'groups of') when engaged in play and encouraged to use numbers to write emergent calculations. Multiplication can be introduced as repeated addition. EG 5+5 = 10. This should be built upon to develop an understanding of multiplication. EG. 2 lots of 5 = 10.



Responds, in practical situations, to questions such as:

How many socks in two pairs?
How many 10p coins are here? How much money is that?

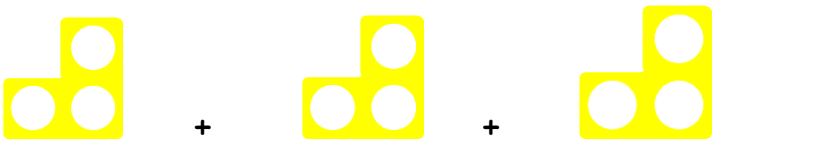
They will use related vocabulary and symbols to describe and record multiplication number sentences.
They will count in steps of 2, 5 and 10.

Y2

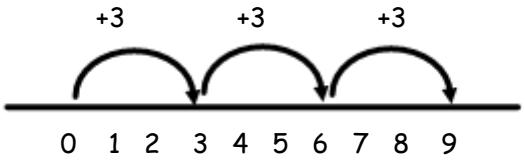
Expectations: Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward.
Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

Multiplication as repeated addition

These visual representations should be used to develop an understanding of multiplication as repeated addition. EG. 3 x 3 (introduced as 3 lots of 3) can be introduced as 3 + 3 + 3= 9.
Numicon, drawings and sketches can be used to support learning of multiplication facts, where appropriate.



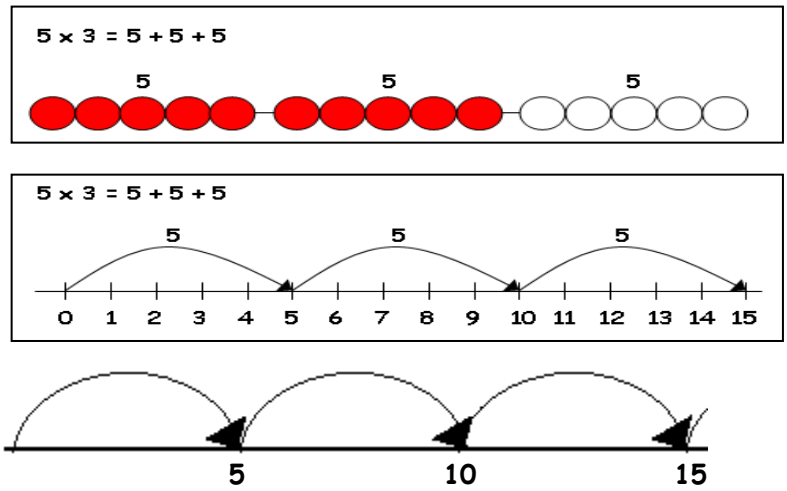
Children's understanding of multiplication as repeated addition can also be developed by using the number line method for addition. This should involve a numbered number line first, with empty number lines introduced at a later stage. EG. 3 x 3 = 3 + 3 + 3



Repeated addition can be shown easily on a bead bar/string and also on a horizontal or vertical number line.

With developing recall of multiplication facts, steps in mental multiplication can be recorded by jumping on an empty number line.

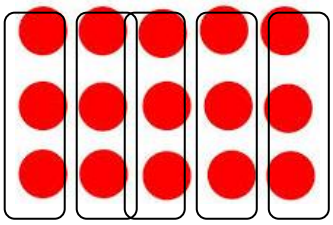
4 x 5 = 20 (*read as 4 lots of 5)*



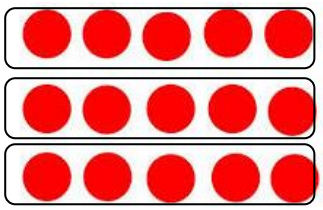
Arrays

Arrays should be introduced as a further method to enable children to see the relationship between corresponding multiplication and division facts.

For example: 5 x 3 or 5 lots of 3 = 15



3 x 5 or 3 lots of 5 = 15



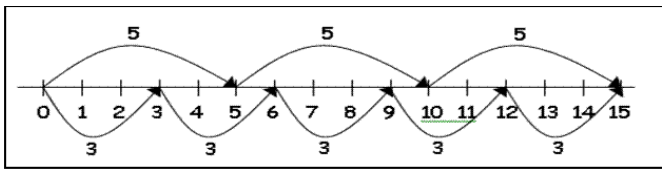
Pupils should also be introduced to the terms 'column' and 'row' at this stage. Arrays can also be continued to demonstrate multiplication as repeated addition. Initially arrays should only be read in rows. Then move into rearranging for the other way of reading the rows. Once the children have got this understanding, it won't be necessary to rearrange them to read in the other direction.

Begin to interpret situations as multiplication calculations, and explain reasoning, for example:

How many wheels are there on three cars?

Commutativity

Commutative law should be introduced and children should know that 3 x 5 is equal to 5 x 3.

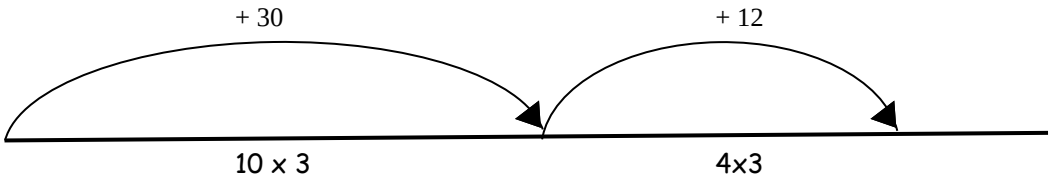


Y3

Expectations: Count from 0 in multiples of 4, 8, 50 and 100.
Solve problems including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

Children use known facts and partitioning to calculate multiplication facts for larger numbers.

14 x 3 (*14 lots of 3)*
= (10 x 3) + (4 x 3) or
= 30 + 12 = 42



Symbols
Using symbols to stand for unknown numbers to complete equations using inverse operations
☐ x 5 = 20 3 x ☐ = 18 ☐ x O = 32

Multiplying by 10, 100, 1000 and the impact on place value.
Children must be taught about the impact of multiplying a number by 10, 100 and 1000. At first, this teaching should be done explicitly and may involve the simplified process through partitioning.

EG. 36 x 10 = 360 It is crucial that children are not allowed to explain this as 'adding' the 0.
36 x 1 = 36

This method can be extended to enable children to multiply multiples of 10, 100 and/or 1000.

EG. 36 x 20 = 720 It is crucial that children are not allowed to explain this as 'adding' the 0.
36 x 2 = 72

EG. 12 x 300 = 3600 It is crucial that children are not allowed to explain this as 'adding' the 0.
12 x 3 = 36

However, this method alone is inefficient in teaching children about the consequence of multiplying by 10, 100 and 1000. To further develop children's understanding, a place value grid should be used.

Pupils should understand that multiplying a number by a number greater than 1 will give an answer that is greater than the original number (EG 7 x 2 = 14). Therefore, a number multiplied by 10, 100 and/or 1000 will move to the left on the place value grid.

EG. 17 X 10 = 170 17 X 100 = 1700 17 X 1000 = 17000

T	U		H	T	U
1	7		1	7	0

T	U		Th	H	T	U
1	7		1	7	0	0

T	U		TTh	Th	H	T	U
1	7		1	7	0	0	0

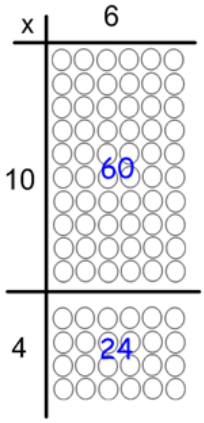
Children may gain access to an understanding of this process by knowing that the 'x' symbol will increase the size of the number and therefore, the digits will move to the left of the place value grid. The number of zeros in the multiplier will then tell the child how many places to the left that they should move the numbers (EG. X 10 = move one place to the left; X 100 = move two places to the left). Children must be reminded to use zeros as 'place holders' when applying this method.

Grid method for multiplication

This method relies on the children's knowledge of multiplication facts, their ability to partition numbers, their understanding of how to multiply multiples of 10, 100, 1000 etc (and also decimals) and finally, the child's ability to add the products together accurately.

Arrays and supporting understanding using expanded multiplication

The previously learned concept of arrays as a visual representation of multiplication facts can be used to enable children to understand the grid method of multiplication.



N.B. It is better to place the number with the most digits in the left hand column of the grid as it is easier to add the partial products and also links to expectations for addition at this stage.

This process can be recorded as an expanded version of short multiplication and should be introduced in Years 3 to enable children to access and use the short method for multiplication by the end of Year 4. EG. 1 digit number x 2/3 digit numbers.

x	8
70	560
3	24
	584

x	3
100	300
40	120
6	18
	438

To enable understanding of what is going on, initially the place value counters can be used to support this method. The children will approximate first e.g. 73 x 8 is approximately 70 x 8, and then will go onto using the grid method to make an accurate calculation.

Y4

Expectations: Count in multiples of 6, 7, 9, 25 and 100.

Solve problems involving multiplying and adding including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Recall multiplication and division facts for multiplication tables up to 12 × 12

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

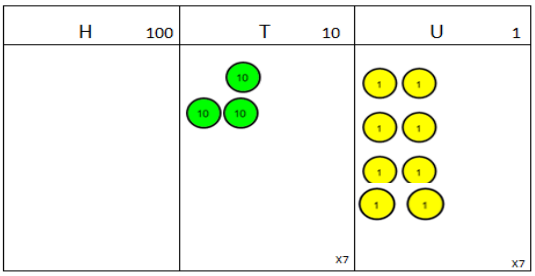
Use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together three numbers.

Recognise and use factor pairs and commutativity in mental calculations

By the end of Year 4, children should be able to use a compact method for short multiplication of HTU x U. (Short multiplication is any number multiplied by a single digit number).

Informal Expanded Written Method
Children have to start with multiplying the ones number.

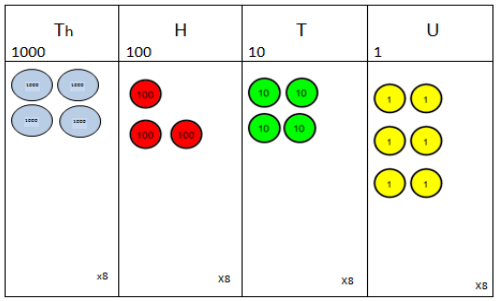
```
  38
 x 7
 56
+210
----
 266
----
```



30 x 7 = 210 8 x 7 = 56

This can be extended up to ThHTU numbers still using the counters and place value to support understanding if needed.

```
   Th H T U
    4 3 4 6
X           8
-----------
        4 8
      3 2 0
    2 4 0 0
  3 2 0 0 0
-----------
  3 4 7 6 8
-----------
```



This expanded method could be used to allow children to understand the step-by-step process for short multiplication. Teachers should note that the aim of using this strategy is to increase understanding and efficiency when using compact short multiplication.

Compact Written Method – short multiplication
Short multiplication should be used to multiply HTU x U by the end of Year 4.

Children MUST begin by multiplying the ones, then tens, then hundreds etc and record the 'carried' numbers underneath the relevant place value grid to compliment the written methods.
The boards and counters can be used as previously but recording is done in the most compact way.

```
  38
 x 7
266
 5
```

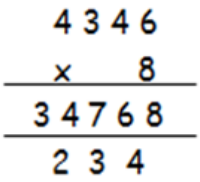
When children are confident using the most compact written method for short multiplication with understanding, along with increased times table knowledge and without needing place value counters, move onto multiplications using larger numbers.

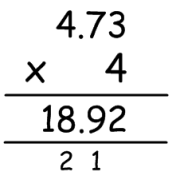
```
  H T U
  1 5 8
X     4
-------
  6 3 2
-------
  2 3
```

Y5

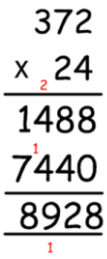
Expectations: Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
Solve problems involving multiplication and division where larger numbers are used by decomposing them into their factors
Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.
Multiply and divide numbers mentally drawing upon known facts.
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
Establish whether a number up to 100 is prime and recall prime numbers up to 19.
Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) .

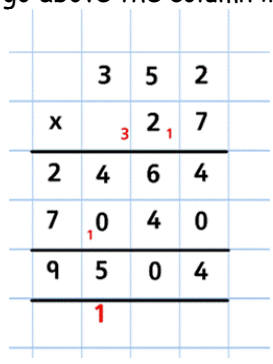
Compact Written Method – short multiplication
Short multiplication should be extended to multiply ThHTU x U and include decimals by the end of Year 5.



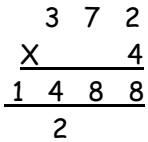


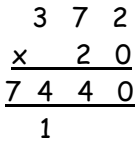
Long Multiplication
This method is an end of Year 5 expectation and an end of phase expectation.
This method can be used to multiply larger numbers by a 2 digit number (larger than 12). Note the positioning of the numbers that are 'carried' - they go above the column into which they will be added.

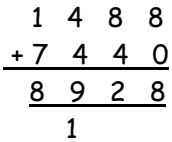




Children may find that they can gain access to this method by partitioning the 2 digit number to create 2 separate 'compact calculations' before adding the totals.
EG. 372 x 24=







However, this method should be reduced to make the method more efficient, with the aim being for all Year 5 and 6 children to use long multiplication as their chosen formal method for multiplication.

Once learned, the long method for multiplication is quick, efficient and reliable. This method compliments the written method for addition, allows children to solve one and two step problems in a range of contexts and is the agreed final stage for the MCSP collaboration, ensuring continuity as children progress into secondary education.

Y6

Expectations: Solve problems involving addition, subtraction, multiplication and division
Perform mental calculations, including with mixed operations and large numbers
Use their knowledge of the order of operations to carry out calculations involving the four operations
Multiply one-digit numbers with up to two decimal places by whole numbers
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Once learned, the long method for multiplication compliments the written method for addition, allows children to solve one and two step problems in a range of contexts and is the agreed final stage for the MCSP collaboration, ensuring continuity as children progress into secondary education.

DIVISION

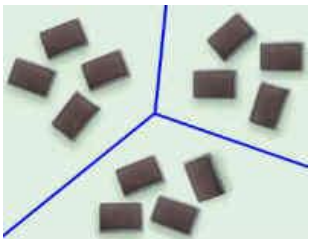
Y1

Expectations: Count in multiples of twos, fives and tens.
Solve one-step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

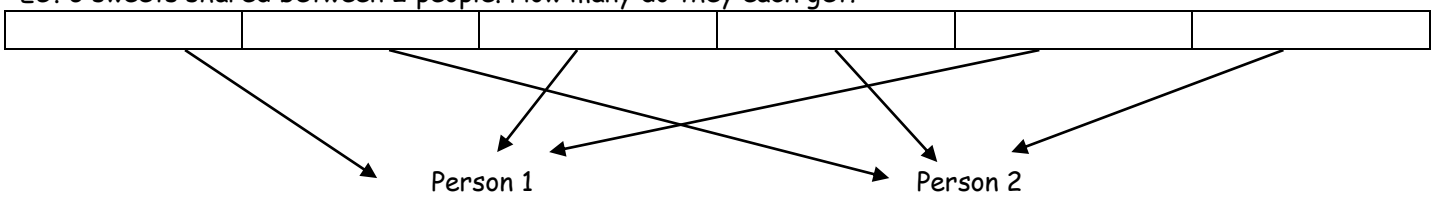
Children should first be encouraged to develop a mental picture of the number system to use for calculation. This may involve the use of Numicon, pictures, visual resources and/or other resources to develop ways of recording calculations. Children should be introduced to range of terms for 'divide' (such as 'share' and 'groups of') when engaged in play and encouraged to use numbers to write emergent calculations. Numicon images and shapes should be included, whenever possible.

Children will understand equal groups and share items out in play and problem solving. Solve problems in everyday life in the classroom, or in role play. Make decisions about what to do. Explain orally and, where appropriate, record the solution in child's own way.

EG. 12 divided by 3 = 4 or *I have 12 pieces of chocolate and share them equally between 3 people. How many pieces of chocolate will each person get?*

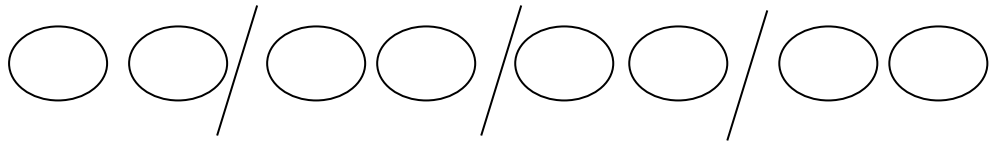


Children will develop their understanding of division and use drawings to support calculation.
EG. 6 sweets shared between 2 people. How many do they each get?



Grouping

'Grouping' can also be used to demonstrate division. EG. *There are 8 sweets. How many people can have 2 sweets each?*



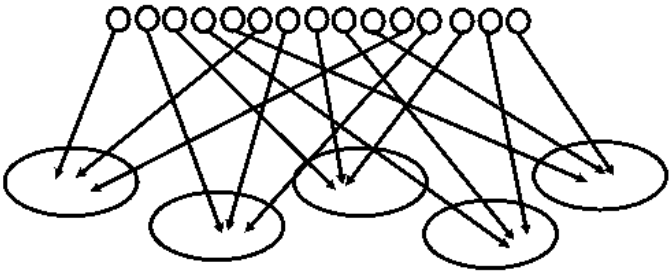
Y2

Expectations: Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward.
Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

Uses related vocabulary and symbols to describe and record ÷ number sentences.
Derives division facts corresponding to the 2 and 10 times tables and begin to for the 5 times table.

Sharing

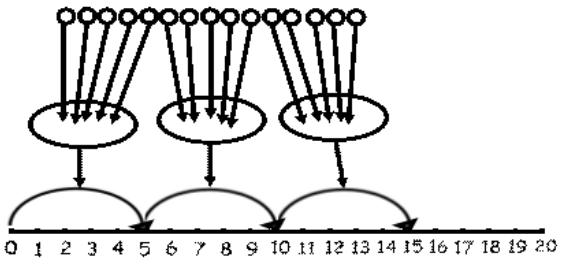
Uses drawings, and at times practical equipment, to share 1 and 2 digit numbers by a single digit number, answering questions such as:
15 shared between 5 15 ÷ 5 =
How many do they get each?



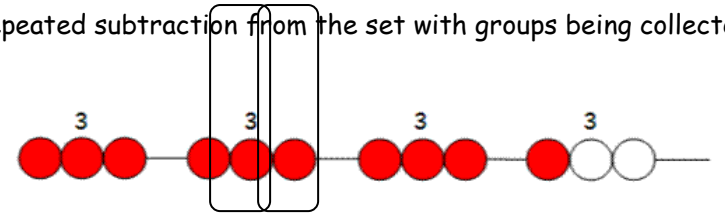
They would get 3 each.

Repeated Subtraction (Grouping)

Uses drawings or the number line and at times practical equipment to repeatedly subtract groups of single digit numbers from 1 and 2 digit numbers, answering questions such as:
How many groups of 5 are there in 15? 15 ÷ 5 =
How many 5s in 15?



Repeated subtraction from the set with groups being collected on the number line or using the bead string.



If using a number line the jumps should be marked underneath as this links to the way in which subtraction is taught.

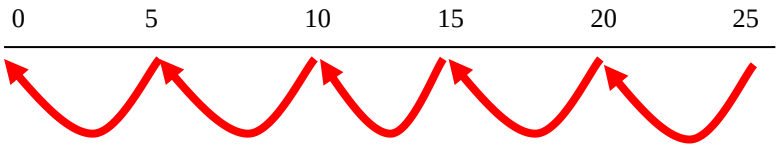
Begin to interpret situations as division calculations and explain reasoning, for example, answer questions such as:

6 sweets are divided equally between 2 people. How many sweets does each one get? **(sharing)**

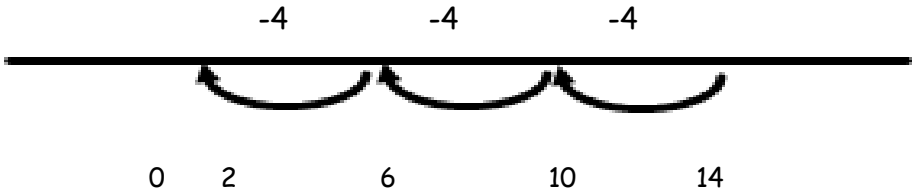
There are 9 apples in a box. How many bags of 3 apples can be filled? **(grouping)**

With developing recall of multiplication facts, steps in mental division can be recorded by jumping on an empty number line forwards or backwards.

25 ÷ 5 = 5



Children should also use this approach to find answers to calculations involving remainders.
EG. 14 divided by 4 = 3 r 2



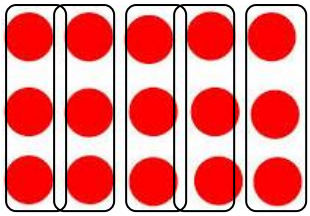
Children should also be using missing number sentences for instance:
26 ÷ 2 = 24 ÷ = 12 ÷ 10 = 8
Children should be seeing the link to multiplication facts in the tables they have learnt, for instance:
If you know that 9 x 4 = 36 then 36 ÷ 4 = and 36÷ = 4

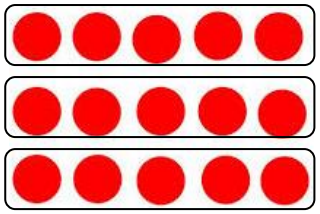
Children should begin to solve a wider range of real life problems and attach understanding to what the remainder means in the context. e.g.
26 friends want to play five a side football.. egg boxes.. etc.

Halving : partition when halving , half the tens and ones separately, then recombine: Halve 36.
Half 30 = 15
Half 6 = 3
= 15 + 3
= 18

Arrays
Arrays should be introduced as a further method to enable children to see the relationship between corresponding multiplication and division facts.

For example: 15 divided by/grouped into 3s = 5 15 divided by/grouped into 5s = 3





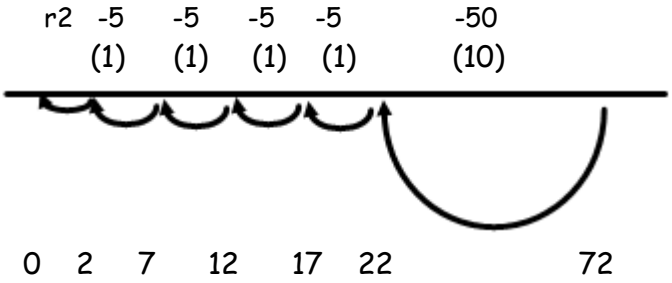
Pupils should also be introduced to the terms 'column' and 'row' at this stage. Arrays can also be continued to demonstrate division as repeated subtraction by taking away groups of the divider.

Y3

Expectations: Count from 0 in multiples of 4, 8, 50 and 100.
Solve problems including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

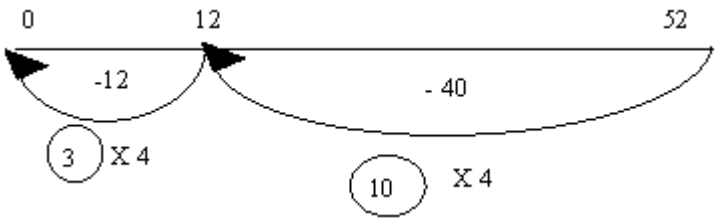
Partitioning

Children should also use the blank number line approach to find answers to calculations involving larger numbers with remainders by combining 'chunking' and repeated subtraction by subtracting multiples of the divisor.
EG. 72 divided by 5 = 14 r 2



For numbers beyond x10 of the divisor use jottings or the number line as a calculating tool.
Jottings: 52 ÷ 4
= (40 + 12) ÷ 4
= (40 ÷ 4) + (12 ÷ 4)
= 10 + 3 = 13

52 ÷ 4 = 13



Dividing by 10, 100, 1000 and the impact on place value.

Children must be taught about the impact of dividing a number by 10, 100 and 1000. At first, this teaching should be done explicitly and may involve the simplified process through partitioning.

EG. 360 div by 10 = 360

This method can be extended to enable children to multiply multiples of 10, 100 and/or 1000.

EG. 240 div by 20 = 12

24 div by 2 = 12

However, this method alone is inefficient in teaching children about the consequence of dividing by 10, 100 and 1000. To further develop children's understanding, a place value grid should be used.

Pupils should understand that dividing a number by a number greater than 1 will give an answer that is less than the original number (EG 12 divided by 2 = 6). Therefore, a number divided by 10, 100 and/or 1000 will move to the right on the place value grid.

EG. 1700 div by 10 = 170 1700 div by 100 = 17 1700 div by 1000 = 1.7

Th	H	T	U		H	T	U		Th	H	T	U		T	U		Th	H	T	U		T	U	.	t
1	7	0	0		1	7	0		1	7	0	0		1	7		1	7	0	0		1	.	7

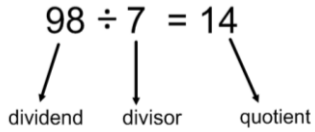
Children may gain access to an understanding of this process by knowing that the 'divide' symbol will decrease the size of the number and therefore, the digits will move to the right of the place value grid. The number of zeros in the multiplier will then tell the child how many places to the right that they should move the numbers (EG. Divided by 10 = move one place to the right; divided by 100 = move two places to the right). Children must be reminded to use zeros as 'place holders' when using this method.

Y4

Expectations: Count in multiples of 6, 7, 9, 25 and 100.
Solve problems involving multiplying and adding including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.
Recall multiplication and division facts for multiplication tables up to 12 × 12
Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths.
Use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together three numbers.
Recognise and use factor pairs and commutativity in mental calculations

PARTITIONING

This leads the children to the more compact standard written method, developing an understanding of its structure and efficiency.



The teacher should model using the correct language and encourage children to use it, e.g. *dividend, divisor, quotient*

Encourage children to develop the skill of 'trapping' the dividend to prepare for partitioning.

72 ÷ 3

Facts list 72 ÷ 3

10 x 3 = 30

20 x 3 = 60 ← 72 'Trap the dividend'

30 x 3 = 90

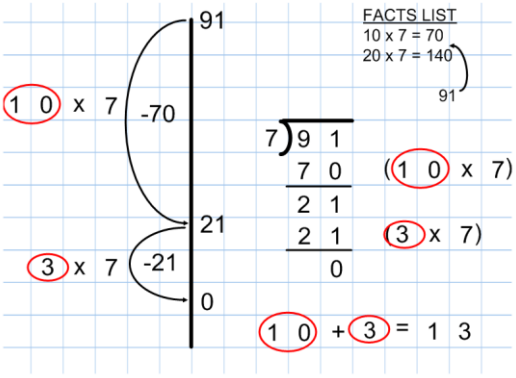
Encourage children to produce a **'FACTS LIST'** using their knowledge of place value and related facts, looking for the two multiples of ten that 'TRAP' the dividend. They should be encouraged to find the largest possible multiple of the divisor and what's left.

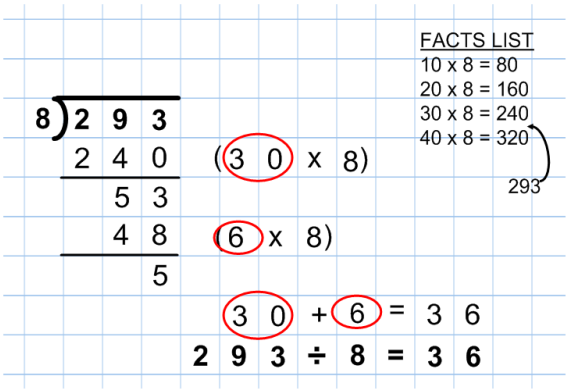
72 is then partitioned into 60 and 12.

72 ÷ 3=
60 ÷3 = 20
12 ÷ 3 = 4

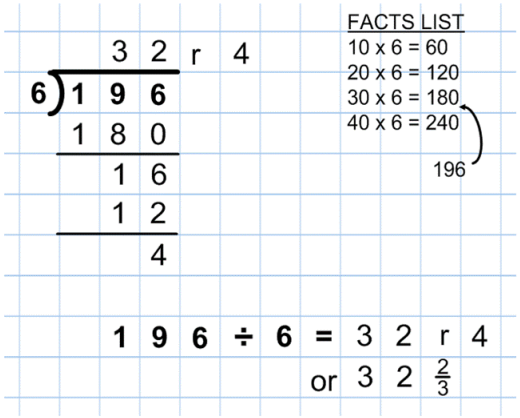
Informal expanded method using chunking
Children use a **facts list** to 'trap the dividend' and help prevent them from creating extremely lengthy chunking calculations.
NB: It may be helpful to introduce this written method alongside a vertical number line as this builds on children's previous learning.

Short division TU ÷ U or HTU ÷ U





This can be expanded to include division calculations including remainders. In a problem solving situation, if the answer is required to be in a decimal format then ensure that the calculations give remainders that are in an easily transferable form.



Y5

Expectations: Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
Solve problems involving multiplication and division where larger numbers are used by decomposing them into their factors
Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.
Multiply and divide numbers mentally drawing upon known facts.
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

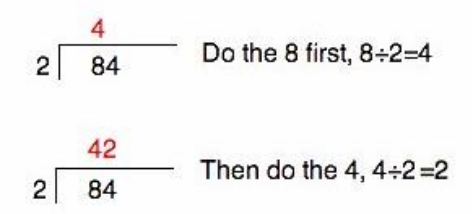
Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
Establish whether a number up to 100 is prime and recall prime numbers up to 19.
Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) .

Compact written method: short division

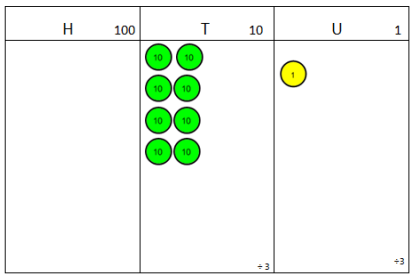
This method relies on the children's knowledge of division facts, their ability to mentally 'chunk lots of', and the success of teacher's modelling of the process and choice of numbers to enable access to the question.

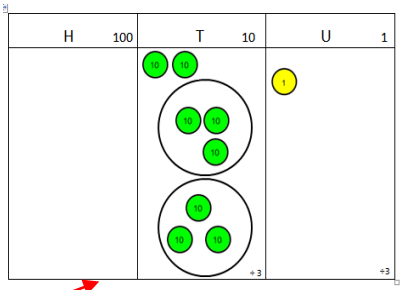
Short division for 2 digit numbers

The previous strategies lead up to the bus stop method of division to divide 2 and 3 digit numbers
Teachers should note that short division or 'the bus-stop method', as it is often called, should only be used when dividing by a one digit number.
Children should start on the left of the calculation and work through the following questions:



TU ÷ U
81 ÷ 3

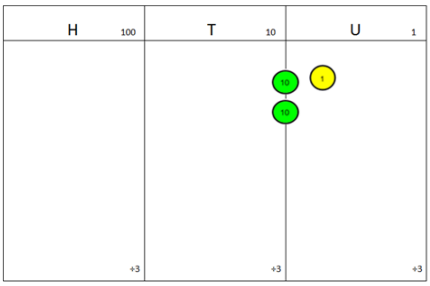




Take groups of 3 tens.
There are 2 groups of 3 tens in 80 with 2 tens left.

T U
2 7
3 | 8 1
2

Children will then mentally work out that there are 7 groups of 3 in 21.

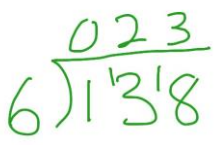


Short division for 3 digit numbers (the same process should be applied to larger numbers that are being divided by a 1 digit number)

Children should start on the left of the calculation and work through the following questions:

- Can I divide the number furthest to the left by the divider? NO. Carry 1 into next PV column.
- Reminder children to write the place holder (0) above.
- Can I divide the next number by the divider? Yes. 2 r1
- Carry the remainder into the next PV column.
- Can I divide the final number by the divider? Yes
- Show your final answer above the 'bus stop'.

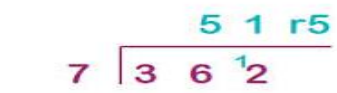
138÷6



Short division and remainders

The same process can be applied to find the answer to division calculations involving remainders.

362 ÷ 7 =



362 ÷ 7 = 51 r5

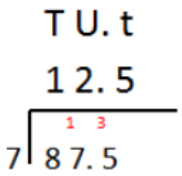
Children should also be shown how to express remainders as fraction and/or a decimal fraction.

For example: 51 and 5/7

Short division and decimals

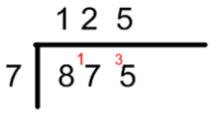
When calculating with decimals and demonstrating understanding to scale up by multiplying by 10 or 100, depending on the number of decimal places; and scale down by the same to give the answer.

If not, the same process should be followed as in short division: There is one group of 7 tens in 80 with one ten left over.



To scale:

E.g. 87.5 ÷ 7 scale up by 10 87.5 x 10 = 875 875 ÷ 7



Scale down by 10 125 ÷ 10 = 12.5

87.5 ÷ 7 = 12.5

Y6

Expectations: Solve problems involving addition, subtraction, multiplication and division

Perform mental calculations, including with mixed operations and large numbers

Use their knowledge of the order of operations to carry out calculations involving the four operations

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Compact method: Long multiplication (dividing by a number with two or more digits)

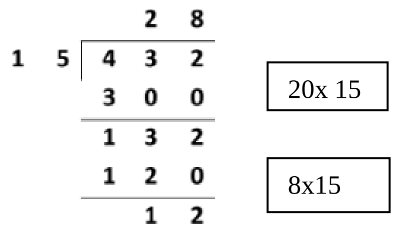
Children must be taught to recognise the need for, and choose this method for dividing by a number with 2 or more digits. This method relies on the children's knowledge of division facts, their ability to mentally 'chunk lots of', and the success of teacher's modelling of the process and choice of numbers to enable access to the question.

Children should become more efficient by subtracting multiples of 10, based on their knowledge of number facts. EG. If I know that 10 x 7 = 70; then I know that 20 x 7 = 140 and 30 x 7= 210.

This can then be applied to the calculation to reduce the number of steps needed to find the final answer and/or remainder.

The 'lots of' are subtracted, with the number of 'lots of' recorded in brackets next to the subtraction calculation. This process is continued until the child can no longer chunk any whole 'lots of' the divider. This number is then recorded as the remainder, next to the sum of the 'lots of'.

432 ÷ 15 becomes



$\frac{12}{15} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

Children should also be shown how to express remainders as fraction and/or a decimal fraction.
For example: 28 r 12 = 28 r 12/15 = 28 r 4/5 = 28.

<u>Long division and 'trapping the dividend'</u>
Children should create a 'facts list' that will help them to become more efficient when 'chunking', so that they do not repeatedly subtract 10 'lots of'. This list should be used to *trap the dividend* and allow children to make an accurate estimate before they calculate.

5 x 15 = 75	
10 x 15 = 150	The dividend has been *trapped* between 20
20 x 15 = 300	and 30 'lots' of; therefore, final answer will
30 x 15 = 450	be between 20 and 30.

It is important that the number of 'lots of' is written first to enable efficient mental addition of these 'lots of' when finding the final answer. EG. 28 r 12.

<u>Long division and decimals</u>
Where children have sufficient knowledge of multiplication and the corresponding division facts, it may be appropriate to introduce long division as a method of dividing larger numbers with decimals.
However, teachers should first ensure that children are able to divide simple decimals by applying their knowledge of number facts.

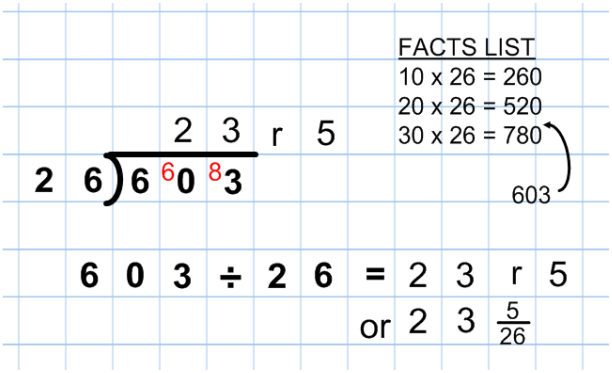
EG. 1.5 divided by 5= 0 . 3

1) Calculate 15 divided by 5 = 3
2) Think about the possible answers. Could 3 be a plausible answer? Why not? Establish that 1.5 is less than 3 and that when dividing, the final answer is less than the number divided. Therefore, 0.3.

Children can then apply this approach to calculate 72.5 divided by 5 =

Long division compact method

The recording can then be reduced to get to the most compact written method.



Once learned, this method for long division is efficient and reliable. This method compliments the written method for subtraction, allows children to solve one and two step problems in a range of contexts and is the agreed final stage for the MCSP collaboration, ensuring continuity as children progress into secondary education.